2. Inference. Naiyāyikas view inference as a knowledge source, a *pramāṇa*, that results in *anumiti*, ‘‘veridical inferential awareness.’’ Inference as a *pramāṇa*, as the psychological process producing that result, is in Sanskrit known by the word *anumāna*.

As already pointed out, a veritable inference, or correct inferring, paradigmatically would be informed by a preceding perceiving, which would be its trigger, so to say, for instance, ‘‘There is smoke on yonder mountain,’’ as known perceptually. But of course it is not just any perceptual cognition that sparks a genuine inference. In the terms of a stock example, a locus or object, a mountain, for instance—called the inferential *subject*—must be perceived as exhibiting one property as invariably related to another. That is, it must be perceived as exhibiting a *prover* property, e.g., smoke or smokiness, in a pervasion relationship with another property called technically the *probandum*, the property to be proved, fire or fieriness. Such a perception is said to be informed by a memory of the pervasion extrapolated from previous experiences. The memory would have resulted from (typically) wide experience of positive and negative correlations (where F, there G, and where no G, there no F). Thus by extrapolation, wherever the prover or prover property, there the probandum or probandum property (‘‘where smokiness, there fieriness’’)—including a perceptually given instance that launches the inference as a *pramāṇa*. The mountain is seen to be qualified by smokiness as further qualified by pervasion-by-fieriness. In other words, the prover property, the smokiness, must be presented as qualified by a pervasion with the probandum property, being-fiery. Throughout New Logic, the nature of the relation called pervasion (*vyāpti*) is the main focus of reflection under the general rubric of inference—pervasion, that is, insofar as knowledge of it is a causal factor in generating inferential awareness. In his inference chapter, Gaṅgeśa considers and rejects more than twenty definitions of pervasion, *vyāpti*. He accepts a total of eight. The upshot is that all things that are qualified by a prover property must also be qualified by a probandum property such that the former is a sign or mark of the latter. (More about pervasions and how they are known just below.)

Gaṅgeśa employs regularly, whatever the topic, a three-part inferential statement form that he assumes his readers can reconstruct as a five-step argument form standardized throughout classical philosophical discourse. He rarely uses the five-step argument form. But it is helpful for learning Nyāya’s logical terminology.

First, the logical terms, which are for the most part common across schools, their technical meanings not being confined to Nyāya:

pakṣa	the inferential subject; the mountain in the stock example
sādhana	the prover or prover property (*hetu* and *liṅga* are synonyms); smokiness
sādhya	the probandum, the property to be proved; fieriness
vyāpti	the inference-grounding pervasion; ‘‘wherever smoke, there fire’’
dṛṣṭânta	a pervasion-supportive example, a basis for extrapolation, typically a locus known to exhibit both the prover and the probandum; a kitchen hearth.

The stock five-step example:

1. That mountain is fiery. (The proposition to be proved, that is, that the *pakṣa*, the inferential subject, is qualified by the *sādhya*, the

probandum.)

2. That mountain is smoky. (The inferential subject is qualified by the prover, that is, the *pakṣa* exhibits the *sādhana*, the prover. This premise is normally to be taken as, in context, perceptually warranted.)

3. Whatever is smoky is fiery. (Or, ‘‘Wherever smoke, there fire.’’ The statement of the general rule or inference-grounding *vyāpti*, pervasion: wherever the prover, there the probandum. This premise is typically warranted because of wide experience of positive correlations, such as smoke and fire in a kitchen hearth—known as an example or *dṛṣṭânta*—and negative correlations, such as absence of smoke on a lake where there is absence of fire.)

4. That smoky mountain falls under the ‘‘whatever’’ of the general rule, ‘‘Whatever . . . ’’ (The mountain falls within the scope of a universal quantifier.)

5. That mountain is fiery. (Conclusion, same as step one, except now proved.)

The three-part statement form:

> (part one: S*a*) An inferential subject *a* is qualified by S a probandum
> (part two: *since* H*a*) *since* that same subject *a* is qualified by H a prover
> (part three: like H*b* and S*b*), like *b* an example.

Since a prover and a probandum are mentioned in any three-part statement, an inference-grounding pervasion is implied: wherever the prover, there the probandum. Gaṅgeśa’s use of the three-part form is signaled in translation by italicization of the word *since*—a convention adopted throughout.

Logical notation so facilitates comprehension that a few symbols are practically indispensable in rendering certain passages of the inference chapter, including stretches of text here. I shall employ symbols mainly as expositional aids, though I shall also try to explain everything without symbols. Pervasion can be interpreted with the tools of set theory, and a logical notation used that, if not perfect, does tightly correspond with inferences expressed in Sanskrit.

The reconstruction below is not adequate to every dimension of Gaṅgeśa’s thought about pervasion. In particular, the predication relation, not expressed below by a sign, can vary, and needs to be indicated in some contexts where special inference rules have to be invoked to handle peculiar types of object. Furthermore, Gaṅgeśa interprets pervasions ontologically as causal relations, which divide into three basic types having some logically important differences. The difference between a factor necessary for an effect and a set of factors together sufficient is also crucial. (On these points, see below, pp. 99 and 116.)

How the conditionals used below (represented by arrows) should be interpreted to tie up with Gaṅgeśa’s causal outlook is a live question, especially since symbolic representation of talk of causal relations in English and other natural languages is a live issue in contemporary philosophy. One candidate is to add a necessity operator to the universally quantified expression: a ‘$\Box$’ would be prefixed to line 2 that would be interpreted within a standard system of modal logic. Another candidate would be to use a counterfactual conditional arrow *within* the line, ‘$\Box\!\!\rightarrow$’ (a move especially attuned to Gaṅgeśa’s identification of ‘‘dubious’’ *upādhi*s: see below, pp. 110ff). However, since possibility and necessity do not figure on the surface of Gaṅgeśa’s schema such modal interpretations would be motivated more by what Gaṅgeśa is reasoning about than by his form of reasoning. And the representation would still be inadequate to all the thought about causality. Furthermore, the schema does not do justice to the inductive character of knowledge of pervasion as a precondition for the operation of inference. Extrapolation failure is perhaps the best way to see how an *upādhi* undermines a pseudo-inference, as will be explained. Thus how best to represent Gaṅgeśa's system of inference is an unfinished scholarly task. The representation provided, I repeat, should be considered principally a pedagogical prop. (See also Figure A, below, p. 23.) {UNFORNUTATELY, THIS AND OTHER FIGURES ARE NOT E-AVAILABLE.}

Schematically, a pervasion *by* a probandum S *of* a prover H suggests a conditional relation: if an H, then an S. If smoke is on a mountain, then fire is on that mountain taken as inferential subject, because there is a general relation, captured by a universally quantified conditional expression, between smoke and fire. Thus a reconstruction of the basic inferential pattern would be (compare the drawings on p. 23):

1. H*a* (the subject, *a*, is qualified by H, the *sādhana* or *hetu* property, as represented by the ''x'' in the lower drawing of Figure A). And

2. (x) (Hx → Sx) (wherever the H property, there the S property, as represented by the shading in both drawings in Figure A). Therefore,

3. S*a* (the subject, *a*, is qualified by S).

This pattern of reasoning, though deductive, is related to an inductive procedure, a way of discerning pervasion with reference to cases other than that at issue: putatively similar instances, *sapakṣa*, which are known to exhibit the probandum and thus should also, at least in one case, exhibit the prover (e.g., H*b* and S*b*)—and putatively dissimilar instances, *vipakṣa*, which are known not to exhibit the probandum and should lack the prover, too (e.g., ~S*c* and ~H*c*). The example, *dṛṣṭânta*, is normally a single instance of *sapakṣa* where the prover also occurs (H*b* and S*b*), its citing thus indicating that there is inductive support. In other words, by reference to cases that are *sapakṣa*—i.e., known instances of the probandum—where the prover is also found, we marshal positive evidence supporting an inference-underpinning pervasion (the method of agreement, *anvaya*, ''positive correlation''). And by reference to cases that are *vipakṣa*—i.e., known instances of the absence of the probandum—we garner further support by not finding the prover (the method of disagreement, *vyatireka*, ''negative correlation''). Citing an example is shorthand to indicate that such a procedure supports assertion of a *vyāpti*, ''pervasion'' or general rule. (The pervasions that are known only positively, called the ''only-positive,'' *kevalânvayin*, and those that are known only negatively, called the ''only-negative,'' *kevala-vyatirekin*, are treated in their own sections of the inference chapter, and the peculiar epistemic problems involved with inferences based on pervasion knowledge of these two types will not be a concern here.)

The inductive procedure is significant for the wider theory of justification more generally and for the logic of the *upādhi* in particular. Each line of the reconstruction above should be prefixed by the epistemic operator ''(W)''—to be read, ''A subject S has warranted cognition that''—and the entire argument can be read, ''If S has warranted cognition that both H*a* and (x) (Hx → Sx), then, *provided that* (a further condition has to be added) S comes to see (by a process called *parāmarśa*, ''reflective grasping of the connection between H*a* and (x) (Hx → Sx)''), S has warranted cognition that S*a*.

1. (W) H*a*
2. (W) (x) (Hx → Sx)
3. (W) S*a*, *provided that* an appropriate *parāmarśa* has occurred.

Gaṅgeśa devotes a long section of his inference chapter to the nature of the required *parāmarśa*, which he sees as, psychologically speaking, the proximate cause of the drawing of an inferential conclusion, of the arising of a veridical inferential awareness (*anumiti*). Thus being-warranted is not closed under deduction considered in abstraction from the psychological process of ‘‘reflection,’’ *parāmarśa*. But through that process, warrant—or ‘‘certainty’’ (*niścayatā*)—passes from premises to conclusion, so to say, and we act unhesitatingly, for example, to put out a fire on yonder mountain. This does not occur, however, if we become aware of an *upādhi*, which would be a blocker of ‘‘reflection’’ through undermining the extrapolation on which such reflection depends. Thus awareness of an *upādhi* is a preventer of inferential awareness, a ‘‘defeater,’’ leading to belief relinquishment by someone who has hitherto not noticed the undercutting condition and who has erroneously arrived at a conclusion preserved in memory.

Taking a wider view, we see that Gaṅgeśa does not think of inferences as bits of reasoning whose correctness can be conclusively determined in isolation, independently of other things known or discoverable. Rather, a judgment of the cogency of a process that is putatively inferential cannot be separated from concern with truth overall. For Gaṅgeśa, apparent inference is ‘‘non-monotonic,’’ defeasible and open-ended, open to correction and support from information gathered from various sources over the course of our lives. Thus is looking for *upādhi*s and finding inferences that are *upādhi*-free said to be an important method of inquiry in certain contexts. And most importantly, in contexts of debates the *upādhi* is an effective tool for refuting putative inferences. Also we can probe the cogency of our own apparent inferences by considering candidate *upādhi*s.

3. The inferential upādhi. The ‘‘undercutting condition’’ is ontologically an object or property that has a certain relation to other objects or properties. Ontologically, there need be nothing peculiar about an inferential *upādhi*, and the inferential usage of the term is distinct from an ontological usage where the same word means ‘‘surplus property,’’ a property that has no place in the traditional categorial scheme. An inferential *upādhi*, in contrast, is only called an *upādhi* within an inferential context. This point is so thoroughly assumed by Gaṅgeśa and other Naiyāyikas that typically they say that an *upādhi* blocks inferential awareness whereas the correct view is that *awareness* of an *upādhi* does so. Ontologically, the properties or things that serve as *upādhi*s in an inferential context exist independently of cognition. They are out there in the world whether or not anyone would make an inference. They get their epistemic value from an epistemic point of view. They are thus like the epistemic ‘‘excellences’’ discussed above; in fact, they count as epistemic ‘‘faults,’’ since, to repeat, awareness of an *upādhi* prevents a veritable inference considered as a psychological process resulting in veridical inferential awareness. But since even an uncognized *upādhi* means that an apparent inference would go wrong that proceeded on the basis of an apparent pervasion that would not have been assumed if there had been awareness of the *upādhi*, Gaṅgeśa’s realist usage (and dropping of the word ‘‘cognized’’) is not so sloppy as it might seem. Our own usage will tend to follow his lead, though not invariably: sometimes we say ‘‘cognition of an *upādhi*’’ where he would say only ‘‘*upādhi*.’’

An early and *standard* definition (D^s) is found in the texts of several schools. It sets out two conditions. An *upādhi* is:

(D^s) A property U such that

(1) U pervades the probandum S (i.e., anything that is an S is a U), and

(2) U does not pervade the prover H (i.e., there is something that is an H but not a U).

In symbols (see also Figure B, p. 23):

1. $(x)\ (Sx \rightarrow Ux)$ (if something is an S, then it is a U; this is represented by the shading in both drawings of Figure B) and

2. $(\exists x)\ (Hx \cdot \sim Ux)$ (there is something that is an H but not a U; this is represented by the ''x'' in the lower drawing in Figure B)

The two conditions being met, it would follow that there is something that is an H but not an S,

3. $(\exists x)\ (Hx \cdot \sim Sx)$ (something is an H but not an S)

and thus that an original apparent inference—*a* is an S, *since a* is an H (and every H is an S)—fails. In other words, the *upādhi* undercuts the apparent inference by showing that the required pervasion and entailment does not hold:

4. $\sim (x)\ (Hx \rightarrow Sx)$ (that every H is an S is false; compare the lower diagrams of Figures A and B)

{FIGURES A - D ARE NOT E-AVAILABLE.}

Again, the S's *pervading* the H is supposed to be the crucial enabling condition for an original apparent inference, such that a current perceptually given H would be an S on the condition that the inferrer has extrapolated the relationship based on solid evidence. Pervasion is sometimes expressed as S-*hood*'s pervading H-*hood*, a nicety we may ignore, as per Gaṅgeśa's own usual practice, the point being that all our evidence points to every particular H being also an S (relations between universals in Nyāya are cashed out extensionally, in terms of their instances). The Venn diagrams below represent the pervasion relationship by the shading of all the H circle that does not overlap with the S, meaning that that there are no Hs that are not Ss. An *upādhi* shows that the S's pervading the H does not obtain. The ''x'' in Figure B contradicts the shading in Figure A.

To run through the terms of a stock example, an occurrence of fire does not guarantee an occurrence of smoke. Wet fuel is an *upādhi* for an apparent inference of smoke from a cognition of fire. All occurrences of smoke are occurrences of wet fuel (U pervades S), and there is fire in some cases, for instance, molten metal, where there is no wet fuel (U does not pervade H). Fire is thought to occur in molten metal (accounting for the heat). Thus, (an awareness of) an *upādhi* defeats an apparent inference, since it would show that there is an occurrence of the prover where the probandum does not occur, and that might well be the case at issue.

One translation is therefore ''defeater'' in English. There are, however, other kinds of epistemic defeaters, perceptual and testimonial defeaters that we shall not be concerned with here. (''Defeater'' seems best to translate Sanskrit *bādhaka*, and within Nyāya typology an awareness of an *upādhi* is indeed a *bādhaka* but, to repeat, there are ''defeaters'' of other types, too.) There are also other kinds of inferential defeaters: being subject to an *upādhi* is hardly the only way that an apparent inference can go wrong, and awareness of an *upādhi* is not the only way to know that an apparent inference is fallacious. ‘‘Deviation,’’ for example—defined as ‘‘being aware of a counterexample, an instance of the prover where the probandum does not occur’’—is another inferential defeater, an ‘‘apparent (but false) prover,’’ *hetv-ābhāsa*; the *upādhi* is another.

The English term ‘‘undercutting condition’’ is chosen for reasons that will be laid out through the remainder of this introductory discussion—and indeed, in a sense, to the very last page of the book. The challenge is to coin a technical term that (1) gets its precise definition from its systematic usage in Nyāya epistemology (as does ‘‘inferential awareness,’’ ‘‘pervasion.’’ and other ‘‘technical terms,’’ *saṃjña*) but also (2) suggests on its face roughly that role (as the technical use of ‘‘inference’’ is continuous with its everyday employment, both in Sanskrit and English) and (3) connects up somehow connotatively with Nyāya’s ontological *upādhi* and other systematic usages outside of Nyāya-Vaiśeṣika (the term *upādhi* gets also a systematic use in Vedānta and other schools of classical philosophy).

First, then, let us try to delineate more sharply the technical sense of the term—beyond the standard definition moving closer to Gaṅgeśa’s sense. The *upādhi* resembles the ‘‘undercutting defeater’’ as introduced to modern analytic epistemology by John Pollock, *Contemporary Theories of Knowledge* (1986) and in other works. Pollock defines a defeater as follows (p. 38):

> If P is a prima facie reason for S to believe Q, R is a *defeater* for this reason if and only if R is logically consistent with P and (P&R) is not a reason for S to believe Q.

And an undercutting defeater (p. 39):

> If P is a prima facie reason for S to believe Q, R is an *undercutting defeater* for this reason if and only if R is a defeater (for P as a reason for S to believe Q) and R is a reason for S to deny that P would not be true unless Q were true.

To see how far Nyāya’s inferential *upādhi* may be assimilated to these ideas, let us look again at the inductive basis of inference. A subject T prior to becoming aware of an *upādhi* has extrapolated, let us assume, a putative pervasion relationship from a finite set of correlations by him known and not at issue:

H*a* and S*a*
H*b* and S*b*
H*c* and S*c*
. . .

T has in addition to these positive correlations no knowledge of a counterexample (something H but not S), though, let us say for simplicity’s sake, he does know of some things that are S but not H (being-an-H and being-an-H are not co-extensive). Furthermore he has the following negative evidence (though this is also not strictly essential):

~S*d* and ~H*d*
~S*e* and ~H*e*
~S*f* and ~H*f*
. . .

On the basis of all this, T extrapolates: being-an-H is pervaded by being-an-S. T then comes to know by perception that something is, well, not simply an H (H*g*) but an H-qualified-by-S-pervasion, which we shall abbreviate as H*. H**a* is, however, not a *prima facie* but a *conclusive* reason to believe S*a*. To keep in mind the defeasible character of reasoning we have to see T's ''prover'' as not the result of extrapolation, H**a*, but as *a*'s being H as known to be correlated with S through an inductive set of evidence (as above), H^i. With respect to the *prima facie* reason $H^i a$, T's becoming aware of an *upādhi* would, in Pollock's terms, undercut H^i. H**a* would be shown to be false by an *upādhi*—and indeed one way of looking at an *upādhi* is that it shows ''deviation''—but $H^i a$ would be undercut. Awareness of an *upādhi* would mean that T has become apprised of another evidence base (which would be in terms of positive correlations):

S*h* and U*h*
S*i* and U*i*
S*j* and U*j*
. . .

T would have in addition to these positive correlations no knowledge of a counterexample (something S but not U), and, let us say, he does know of some things that are U but not S (being-an-H and being-an-H are not co-extensive, though, as we shall see explicitly stated in Gaṅgeśa's text, they could be co-extensive and there be an *upādhi* nonetheless). We may also imagine that T has negative evidence correlating, with no counterexamples, things ~S and ~U. On the basis of all this, T extrapolates: being-an-S is pervaded by being-an-U. Furthermore, T knows that something is an H but not a U:

$(\exists x)\ (Hx \text{ and } {\sim}Ux)$

T's awareness of these two conditions together—U*, or more precisely, as we shall see, U^i, along with the U's not pervading the H, conditions difficult to combine in a single formula though Gaṅgeśa somehow manages—would be, again in Pollock's terms, R, an *undercutting defeater*, thus blocking or defeating T's warrant for the otherwise inferentially based belief (S*a*).

For example, one would not infer that something is smoky on the basis of its being fiery once one was aware of wet fuel as an *upādhi* (the fire might be in molten metal where there is no wet fuel). Cognition of the *upādhi* undercuts the warrant for the belief that an inferential subject exhibits a probandum by undercutting, so to say, the extrapolative basis on which the proper generation of such a belief would depend, to mix the terms of the two systems. An *upādhi* is not a counterexample, which is, in Nyāya, a separately analyzed ''apparent (but false) prover'' or fallacy. But an *upādhi* entails a counterexample, or, more precisely, at least gives rise to warranted suspicion of a counterexample such that S*a* may not be concluded. There are some fine points here that we will ignore for the moment concerning what Gaṅgeśa calls the ''dubious *upādhi*'' (*saṃdigdhôpādhi*)—which does not strictly entail a counterexample but which undermines an apparent inference nonetheless, as we shall see.

Pollock contrasts an undercutting defeater with a rebutting defeater, which would be a reason to believe not-Q, as opposed to a reason not to believe Q on the basis of P. And, indeed, in some cases an *upādhi* can be viewed as a rebutter. But its primary epistemological employment is as an undercutter. Admittedly, the word ''undercutter'' is a bit too generic since other fallacies also undercut in Pollock's sense. The two systems cannot be assimilated perfectly, and, below, I explain why I add the word ''condition'' to differentiate the *upādhi* from some of its closer kin.

Now, Gaṅgeśa has his *pūrvapakṣin* bring up at the beginning of the *upādhi* treatment a decisive objection to the standard definition, D^s (see above). The argument is that D^s is too narrow, failing to count as an *upādhi* a property that does not pervade a probandum except in conjunction with the putative prover, although such a property would indeed undercut one's inferential warrant in the same way as in other examples. (See Figure C.) The example (cf., the translation below, p. 38) is digestion-of-certain-vegetables (U) with respect to an original apparent inference that runs: ''The child to be born (*a*) will be dark-complexioned (S), *since* it is (H) Mitrā's child. (''Mitrā'' is a woman's name.) Here clearly the proposed *upādhi* does not pervade the ''pure'' probandum, all things dark-complexioned. The mother's digestion of certain vegetables (the *upādhi*, U) does not occur in the case of unbaked pots, for example, which exhibit the particular color specified, ''dark,'' *śyāma*. But it does pervade, or might well pervade, we are to suppose, those things that are both dark-complexioned and occurrences of the prover (H), namely, being-a-child-born-to-Mitrā. We are to imagine that Mitrā, who is pregnant, has several other children and that they are all dark-complexioned. In other words, we are to imagine that someone is purporting to infer on the basis of the fact that all of Mitrā's children so far have been dark-complexioned that the child to be born will be dark-complexioned. The undercutting condition U might be a causal factor such that if Mitrā had not followed her vegetable diet her children would not have been dark-complexioned, and if she has not eaten those vegetables, whatever they are, during her pregnancy this time the child will not be dark-complexioned. In that case, digestion-of-certain-vegetables (U) would pervade the subset (H. S) of the probandum set (S) that is specified by the putative prover (H), being-a-child-of-Mitrā. H may well reflect *a* causal factor, but it alone would not be sufficient to secure S, being-dark-complexioned. Thus the original apparent inference is undermined.

The pared-down *upādhi* pervasion would indeed show deviation of an original prover from its probandum. Again letting ''H'' stand for the prover, ''S'' for the probandum, and ''U'' for the *upādhi*, we are given—through extrapolation—that everything that is both an H and an S is a U, and some H are not U. With this information it is provable that some H are not S. (See also Figure C.)

1. (x) ((Hx. Sx) → Ux) (The undercutting condition U pervades the probandum S as delimited by the prover H; this is represented by the shading in both drawings of Figure C.)
2. (∃x) (Hx. ~Ux) (The undercutting condition U fails to pervade the prover H; this is represented by the ''x'' in the lower drawing of Figure C.)

Therefore, as with D^s,

3. (∃x) (Hx. ~Sx). (There is something that is an H but not an S.)

And so fails an original apparent inference (false but looking good to those ignorant of the *upādhi*) based on the presumption of S pervading H (compare the lower diagrams of Figures A and C). {FIGURES A - D ARE NOT E-AVAILABLE.}

After a long section of further scrutiny of the standard definition and various substitutes, Gaṅgeśa proposes a new definition that will handle this and similar cases and thus not be too narrow. We shall examine that definition—along with others that are exact, or close, equivalents—in comments later, pp. 80ff. For the present, note only that it is our reading that Gaṅgeśa achieves a generalization of the logic of the Mitrā case (as represented by Figure C) without falling into the oppositive error and letting too much in. (Also note that Gaṅgeśa's treatment of what he calls ''dubious undercutters'' (*saṃdigdhôpādhi*) forces a modal interpretation not captured by

Figure C, as will be argued in comments to passages in Gaṅgeśa's *siddhânta*, below, pp. 110ff. After all, the exact causal status of eating the vegetables is not known precisely, as Gaṅgeśa will explain, but the apparent inference to Mitrā's giving birth to another dark-complexioned child is undercut nonetheless.)